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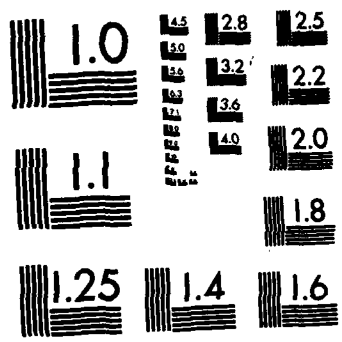

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DIGITAL DISKS

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ABSTRACT

Geometric properties of digital disks are discussed. An algorithm is presented that determines whether or not a given digital region is a digital disk.

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1. Introduction

Shape recognition is a basic part of image processing and pattern recognition. Elementary shapes in the 2D Euclidean plane, such as polygons and conic sections, are well defined and understood. However, the objects in digital image processing and pattern recognition are not continuous objects but their digital representations. The shapes of sets of digital points are not well defined and not well understood.

The connectivity and convexity of sets of digital points have been studied extensively [2,3,6,8]. These are geometric properties closely related to the shapes of sets of digital points. The convex polygonality of sets of digital points is discussed in [4]. This paper treats the shape of a disk for sets of digital points. We discuss criteria for saying that a set of digital points has the shape of a disk; what are the geometric properties that are satisfied by digital disks; and how to determine whether or not a given set of digital points has the shape of a disk.

A measure of the circularity of sets of digital points is discussed in [1]. It gives a parameter that indicates the closeness of a set of digital points to a digital disk. In [5], sets of digital points which are digitizations of circles are characterized. Also, an algorithm for a deterministic tape-bounded array acceptor to determine circularity is presented.

Our characterization of digital disks and algorithm are entirely different from those given in [5]. Further, the algorithm is for general purpose computers.

In the next section definitions and a previously known result that is useful in this paper are given. Simple geometric characterizations of digital disks are discussed in Section 3. Section 4 derives further characterizations of digital disks that lend themselves to development of an algorithm to determine whether or not a digital region is a digital disk. The algorithm is presented and its complexity analyzed in the same section.

2. Definitions

The set of all points in 2D Euclidean space is denoted by E. The set of all points in E with integer-valued coordinates is denoted by D. A point in D is called a digital point. Let S be a set of digital points. Then $\overline{S}$ denotes the complement of S in D. A point of S is an interior point if all of its four 4-neighbors [7] are points of S. A point of S is a boundary point if it is not an interior point.

Digital region

A digital region S is any finite subset of D which is 4-connected [4].

Digital image of a region

Consider a region p in E. A set S in D is said to be the digital image (or simply, image) of p, and p a preimage of S, if

(i) $S \subseteq p$ and

(ii) $\overline{S} \cap p = \emptyset$.

We denote the image of p by I(p).

Digital disk

A digital region Q is a digital disk if there is a disk q whose digital image is Q. (See Figure 1.)

Let q be a disk. The boundary (circumference) of q is denoted by ∂q and the center by c(q). If x and y are points on ∂q,

the subset of ∂q obtained by moving from x to y clockwise is denoted by $\partial q(x,y)$. Thus the union of $\partial q(x,y)$ and $\partial q(y,x)$ is ∂q. Given any pair of points x,y in E, we denote the line segment between the two by xy. Given three points x,y, and z, the angle measured clockwise from xy to xz is denoted by yxz. Given two points x,y on ∂q, q(x,y) denotes the subset of q whose boundary is the union of xy and $\partial q(x,y)$. Let u be a point in E. The distance from u to ∂q, denoted dist(u,∂q), is defined as follows: The line that passes u and c(q) intersects ∂q at two points. Let the one nearest to u be v. Then dist(u,∂q) is the length of uv. (See Figure 2.)

Convex digital region

A digital region S is said to be convex if there is a convex region p whose digital image is S.

Given a digital region S, H(S) denotes its convex hull.

A result in [4] which is used in this paper is stated as a lemma below.

Lemma A

A digital region S is convex if and only if H(S) does not contain any point of $\overline{S}$.

The following corollary is immediate from the lemma.

Corollary B

A digital disk is convex.

3. Geometric properties of digital disks

In this section we discuss simple geometric properties that characterize digital disks. These are interesting geometric characterizations, but they do not immediately lend themselves to development of an algorithm that determines whether or not a given digital region is a digital disk. Still, they are a basis from which further characterizations of digital disks are derived that enable us to design such an algorithm.

The first result is on a necessary and sufficient geometric property for a digital region to be a digital disk. The second is on a geometric property which implies that a digital region is not a digital disk.

Before presenting these results, we introduce some notation. Let x,y and z be three distinct points in E. Let $\overline{xy}$ denote the half-line with endpoint x that passes y. We denote by h(yxz) the unbounded subset of E that is obtained by rotating $\overline{xy}$ clockwise to $\overline{xz}$.

Theorem 1. A digital region Q is a digital disk if and only if there is a (Euclidean) disk q that satisfies one of the following:

(i) Every point of Q is a point of q and no point of $\overline{Q}$ is a point of q. Also there are three points d_1, d_2, d_3 of Q on ∂q such that none of the angles $d_1c(q)d_2$, $d_2c(q)d_3$ and $d_3c(q)d_1$ are greater than π.

(ii) Every point of Q is a point of q and there are two points d_1, d_2 of Q on ∂q such that all the digital points on $\partial q(d_1, d_2)$ are points of Q and all the digital points on $\partial q(d_2, d_1)$ are points of $\bar{Q}$. Moreover, the rest of the points of $\bar{Q}$ are points of $\bar{q}$.

Proof: Suppose that given a digital region Q, there exists a disk q satisfying either (i) or (ii). If q satisfies (i), then Q is the digital image of q and hence, is a digital disk by definition. So assume that q satisfies (ii). Let

$$\delta_1 = \min_{d \in h(d_1 c(q) d_2) \cap \bar{Q}} \{\mathrm{dist}(d, \partial q)\} \text{ and}$$

$$\delta_2 = \min_{d \in h(d_2 c(q) d_1) \cap Q - \{d_1, d_2\}} \{\mathrm{dist}(d, \partial q)\}.$$

Let $\delta = \frac{1}{2} * \min\{\delta_1, \delta_2\}$. Then $\delta > 0$, since obviously $\delta_1, \delta_2 > 0$. Let s be the radius of q that bisects the angle $d_1 c(q) d_2$, and c' be the point on s such that the distance between c(q) and c' is δ. (See Figure 3.) We denote by p the disk whose center is c' and whose circumference contains d_1 and d_2. Due to the construction of δ, it is obvious that q-p does not contain any point of Q and p-q does not contain any point of $\bar{Q}$. Thus, every point of Q is a point of p and no point of $\bar{Q}$ is a point of p. Therefore, Q is the digital image of p and hence, is a digital disk. This completes the proof of the "if" part.

Next, suppose that a digital region Q is a digital disk. Then there is a disk p whose digital image is Q. If there are three points d_1, d_2, d_3 of Q such that none of the angles

$d_1c(p)d_2$, $d_2c(p)d_3$ and $d_3c(p)d_1$ are greater than π, then we are done because p satisfies (i). So assume that there are no such three points of Q on ∂p, and consider the following cases:

<u>Case 1</u> There are at least two points of Q on ∂p.

Then there are two points d_1,d_2 of Q on p such that there is no point of Q on $\partial p(d_2,d_1)$ and the angle $d_2c(p)d_1$ is greater than π. (See Figure 4.) Let s be the radius bisecting the angle $d_1c(p)d_2$. Consider point c that, starting from c(p), moves continuously away from c(p) along the radius s. As c moves, consider the disk q whose center is at c and radius is $cd_1(=cd_2)$. As soon as $\partial q(d_2,d_1)$ touches points of Q or $\partial q(d_1,d_2)$ toucches points of $\overline{Q}$ or both, point c stops its movement.

Suppose $\partial q(d_1,d_2)$ touched points of $\overline{Q}$. Note that even if $\partial p(d_1,d_2)$ had points of Q, $\partial q(d_1,d_2)$ has no point of Q. Thus, every digital point on $\partial q(d_1,d_2)$ is a point of $\overline{Q}$ and every digital point on $\partial q(d_2,d_1)$ (there may not be any) is a point of Q. Therefore, q satisfies (ii) and we are done. So suppose that $\partial q(d_1,d_2)$ did not touch any point of $\overline{Q}$ but $\partial q(d_2,d_1)$ touched points of Q. If there is a point d_3 of Q on $\partial q(d_2,d_1)$ such that the angles $d_2c(q)d_3$ and $d_3c(q)d_1$ are both less than π, then q satisfies (i) and the proof for case 1 is complete. Assume that there is no such point of Q on $\partial q(d_2,d_1)$. Let d_3 be the point of Q on $\partial q(d_2,d_1)$ such that the angle $d_3c(q)d_1$ is

smallest, which still is larger than π. Now we are back to the original condition of case 1 except that now d_3 replaces d_2. We repeat the process until the new disk q satisfies either (i) or (ii). This process terminates, since $\partial q(d_1,d_2)$ eventually touches a point of $\overline{Q}$. This completes the proof for case 1.

Case 2 There is one point of Q on p.

Let d_1 be the point of Q on ∂p and s be the radius of p whose endpoints are d_1 and c(p). Consider point c that starting from c(p) moves continuously toward d_1 along s. The point c stops moving when ∂q touches points of Q for the first time, where q is the circle whose center is at c and radius is cd_1. If there are two points d_2, d_3 of Q on ∂q such that none of the angles $d_1c(q)d_2$, $d_2c(q)d_3$ and $d_3c(q)d_1$ are greater than π, then q satisfies (i). If not, this case is reduced to case 1.

Case 3 There is no point of Q on ∂p.

Let q be the largest disk such that its center coincides with the center of p, i.e., c(q)=c(p), and there are points of Q on ∂q. Now the case is reduced to one of the above cases depending on the number of points of Q on ∂q and their relative positions. This completes the proof of the "only if" part.

The proof of the theorem is now complete. □

Theorem 2. Let Q be a digital region and p a disk. If there are two points d_1, d_2 of Q on ∂p such that both subsets $p(d_1, d_2)$ and $p(d_2, d_1)$ contain points of $\overline{Q}$, then Q is not a digital disk.

Proof: Given a digital region Q and a disk p, suppose that two points d_1, d_2 of Q on p are such that points e_1 and e_2 of $\overline{Q}$ are in $p(d_1, d_2)$ and $p(d_2, d_1)$, respectively. If either e_1 or e_2 is a point on the line segment d_1d_2, then there is no disk that contains both d_1 and d_2 but not e_1 or e_2 and so Q is not a digital disk. Hence, assume that neither e_1 nor e_2 is a point on d_1d_2. Suppose that Q is a digital disk. Then there is a disk q whose digital image is Q and in particular, d_1 and d_2 are points of q and e_1 is not. Thus, ∂q intersects ∂p at two points on $\partial p(d_1, d_2)$. (See Figure 5.) Since two circles may intersect at most at two points, $p(d_2, d_1)$ is a subset of q. Hence, e_2 is a point of q, which contradicts the fact that Q is the digital image of q. Therefore, Q is not a digital disk. □

4. Algorithm

An algorithm is presented that determines whether or not a given digital region is a digital disk. Since the results derived in the previous section do not lead to an efficient algorithm, further characterizations of digital disks are needed. A few of them are given as lemmas after introducing some necessary notation.

We represent a polygon by the sequence of its vertices listed in the order of clockwise traversal of its boundary. Consider a digital region Q and its convex hull H(Q). Since H(Q) is a polygon, it may be represented by the sequence of its vertices $(v_1, v_2, \ldots, v_n)$, where the v_i's are obviously points of Q. We denote the set $\{v_1, v_2, \ldots, v_n\}$ of the vertices of H(Q) by P. Let R denote the set of digital points that are boundary points of $\overline{Q}$. (See Figure 6.) Given any pair of two distinct points v_i, v_j of P, $\overrightarrow{v_i v_j}$ denotes the line through the two points with its sense in the direction from v_i to v_j. Let $P(v_i, v_j)$ and $R(v_i, v_j)$ denote the subsets of P and R, respectively, that lie to the left of $\overrightarrow{v_i v_j}$. Thus $P(v_i,v_j) \cup P(v_j,v_i) = P - \{v_i, v_j\}$ and $R(v_i,v_j) \cup R(v_j,v_i) = R - (R \cap \{x \mid x \text{ is on } \overrightarrow{v_i v_j}\})$. (See Figure 6.) Let v be the point of $P(v_i, v_j)$ such that the angle $v_i v v_j$ is not greater than the angle $v_i v' v_j$ for any v' in $P(v_i, v_j)$, and let minang$(P(v_i,v_j))$ denote this angle. That is, minang$(P(v_i,v_j)) = \min\{v_i v v_j \mid v \in P(v_i,v_j)\}$. Similarly, maxang$(R(v_i,v_j)) = \max\{v_i w v_j \mid w \in R(v_i,v_j)\}$.

The digital region Q is convex if it is a digital disk by Corollary B. So it is obvious that the convex digital region Q is a digital disk if and only if there is a disk q that contains the vertices of H(Q) and does not contain any point of $\overline{Q}$. This observation and the results in the preceding section lead us to the following lemmas.

Lemma 3

Given a digital region Q, let v_i and v_j be any two distinct vertices of H(Q). If $\mathrm{maxang}(R(v_i,v_j))+\mathrm{maxang}(R(v_j,v_i)) \geq \pi$, then Q is not a digital disk.

Proof: Let w be a point of $R(v_i,v_j)$ such that the angle $v_i w v_j$ is equal to $\mathrm{maxang}(R(v_i,v_j))$. Denote by q the disk whose circumference is determined by v_i, v_j and w. If w' is a point of $R(v_j,v_i)$ such that the angle $v_j w' v_i$ is equal to $\mathrm{maxang}(R(v_j,v_i))$, then $v_j w' v_i \geq \pi - v_i w v_j$, and w' is a point of $q(v_j,v_i)$. Therefore, v_i and v_j are points of Q on ∂q and $q(v_i,v_j)$ and $q(v_j,v_i)$ each has a point of $\overline{Q}$. Hence, Q is not a digital disk by Theorem 2. □

Lemma 4

Given a digital region Q, let v_i and v_j be any two distinct vertices of H(Q). Suppose that the following conditions are satisified:

(i) $\mathrm{minang}(P(v_i,v_j))+\mathrm{minang}(P(v_j,v_i)) \geq \pi$,

(ii) $\mathrm{maxang}(R(v_i,v_j))+\mathrm{maxang}(R(v_j,v_i)) < \pi$, and

(iii) $\text{minang}(P(v_i,v_j)) > \text{maxang}(R(v_i,v_j))$ and

$\text{minang}(P(v_j,v_i)) > \text{maxang}(R(v_j,v_i))$.

Then Q is a digital disk.

Proof: Assume without loss of generality that $\text{minang}(P(v_i,v_j)) \leq \text{minang}(P(v_j,v_i))$. Consider the following two cases:

Case 1 $\text{minang}(P(v_i,v_j)) + \text{maxang}(R(v_j,v_i)) < \pi$.

Let v be a point of $P(v_i,v_j)$ such that the angle $v_i v v_j$ is equal to $\text{minang}(P(v_i,v_j))$. Then let q be the disk whose circumference contains the points v_i, v and v_j. Since for every v' of $P(v_i,v_j)$, the angle $v_i v' v_j$ is not less than $\text{minang}(P(v_i,v_j))$, v' is a point of q. If a point x lies to the left of $\overrightarrow{v_j v_i}$ and the angle $v_j x v_i$ is larger than or equal to $\pi - \text{minang}(P(v_i,v_j))$, then it is a point of $q(v_j,v_i)$. Since $\text{minang}(P(v_j,v_i)) \geq \pi - \text{minang}(P(v_i,v_j))$, the angle $v_j v' v_i \geq \pi - \text{minang}(P(v_i,v_j))$ for every v' of $P(v_j,v_i)$. Hence, every point of $P(v_j,v_i)$ is a point of $q(v_j, v_i)$. Therefore, every vertex of H(Q) is a point of q and thus Q is contained in q. Now we show that no point of $\overline{Q}$ is a point of q. For any point w of $R(v_i,v_j)$, the angle $v_i w v_j$ is less than or equal to $\text{maxang}(R(v_i,v_j))$ which is less than $\text{minang}(P(v_i,v_j))$. So w is not a point of $q(v_i,v_j)$. Since w is obviously not a point of $q(v_j,v_i)$, it is not a point of q. Consider a point of $R(v_j,v_i)$, denoted w. The angle $v_j w v_i$ is not greater than $\text{maxang}(R(v_j,v_i))$, which in turn is less than $\pi - \text{minang}(P(v_i,v_j))$. Thus, w is not a point of $q(v_j,v_i)$, and so is not a point of q. We have shown that

every point of Q is a point of q and no point of $\overline{Q}$ is a point of q. Therefore, Q is a digital disk.(It is not difficult to see that q satisfies condition (i) of Theorem 1.)

Case 2 $\text{minang}(P(v_i,v_j))+\text{maxang}(R(v_j,v_i))\geq\pi$.

Let w be a point of $R(v_j,v_i)$ such that the angle v_jwv_i is equal to $\text{maxang}(R(v_j,v_i))$. Consider the disk q whose circumference has points v_i,w and v_j. Since for any point w' of $R(v_j,v_i)$, the angle v_jwv_i is not greater than $\text{maxang}(R(v_j,v_i))$, w' is either a point on $\partial q(v_j,v_i)$ or a point of $\overline{q}$. Let w' be a point of $R(v_i,v_j)$; then the angle $v_iw'v_j$ is less than $\pi-v_jwv_i$, so w' is not a point of q. Next we show that every point of Q is a point of q. If v is a point of $P(v_j,v_i)$, then the angle v_jvv_i is greater than the angle v_jwv_i and hence v is a point of $q(v_j,v_i)$. If v is a point of $P(v_i,v_j)$, then the angle v_ivv_j is greater than or equal to $\pi-v_jwv_i$. Thus, w is a point of $q(v_i,v_j)$. We note that q satisfies condition (ii) of Theorem 1, and so Q is a digital disk. □

Now we are ready to present an algorithm to determine whether or not a given digital region is a digital disk.

Algorithm DIGITAL_DISK(Q)

|| Given a digital region Q, the algorithm determines if Q is a digital disk. If it is, the algorithm prints True and halts, otherwise it prints False and halts. ||

Step 1. Construct the convex hull H(Q).

If H(Q) has a point of $\overline{Q}$ then print (False); stop.

Step 2. Construct the following two sets of digital points:

$P = \{v_1, v_2, \ldots, v_n\}$, where $(v_1, v_2, \ldots, v_n) = H(Q)$.

$R = \{w_1, w_2, \ldots, w_m\}$, the set of boundary points of $\bar{Q}$.

Step 3. For $i=1$ to $n-1$ do

for $j=i+1$ to n do

3.1 evaluate $minang(P(v_i, v_j))$, $minang(P(v_j, v_i))$, $maxang(R(v_i, v_j))$ and $maxang(R(v_j, v_i))$.

3.2 if $maxang(R(v_i, v_j)) + maxang(R(v_j, v_i)) \geq \pi$
then print (False); stop.

3.3 if $minang(P(v_i, v_j)) + minang(P(v_j, v_i)) \geq \pi$,
$maxang(R(v_i, v_j)) + maxang(R(v_j, v_i)) < \pi$, and
$minang(P(v_i, v_j)) > maxang(R(v_i, v_j))$ and
$minang(P(v_j, v_i)) > maxang(R(v_j, v_i))$
then print (True); stop.

Step 4. Print (False); stop.

Suppose that the digital region Q resides in a set of $N \times N$ digital points. We assume that the region is represented by its run length code [7].

Theorem 5. Algorithm DIGITAL_DISK determines whether or not a given digital region is a digital disk requiring $O(N^3)$ time and $O(N)$ work space.

Proof: For a digital disk Q, the condition for q in Theorem 1 implies and is implied by the condition for q in Lemma 4. Thus, the correctness of algorithm DIGITAL_DISK is immediate from Theorem 1, Lemma 3 and Lemma 4.

The work space needed by the algorithm is for H(Q)(=P), R and a few temporaries. Since P has at most 2N elements and R at most 4N elements, O(N) space is all that is required.

The steps 3.1, 3.2 and 3.3 are executed at most $4N^2$ times because n is at most N. The execution time of Step 3.1 is O(N) because at most 6N angles need be evaluated. Steps 3.2 and 3.3 each take constant time. Therefore, the time complexity of the algorithm is $O(N^3)$. □

5. Conclusion

A definition of digital disks is given which is consistent with the definition of digital convexity and digital polygonality. With this definition, we were able to derive a few simple geometric properties that characterize digital disks. Moreover, an algorithm to determine digital disks was developed from these characterizations. The algorithm is conceptually simple and very easy to implement. Its $O(N^3)$ time complexity, however, seems rather excessive, and it is not likely that $O(N^3)$ is the lower bound.

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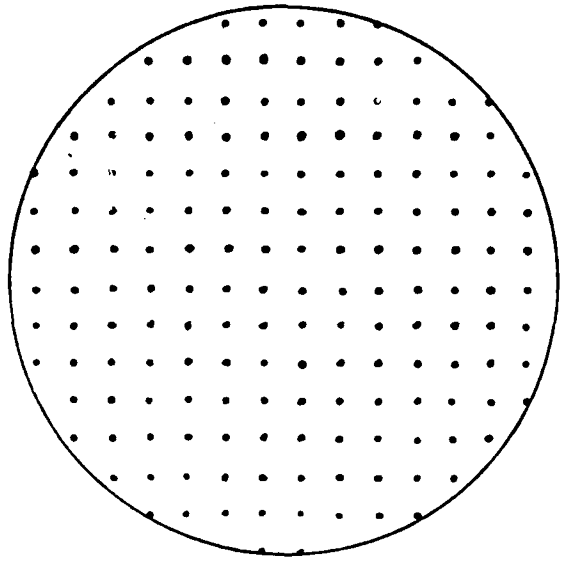

Figure 1. A digital disk.

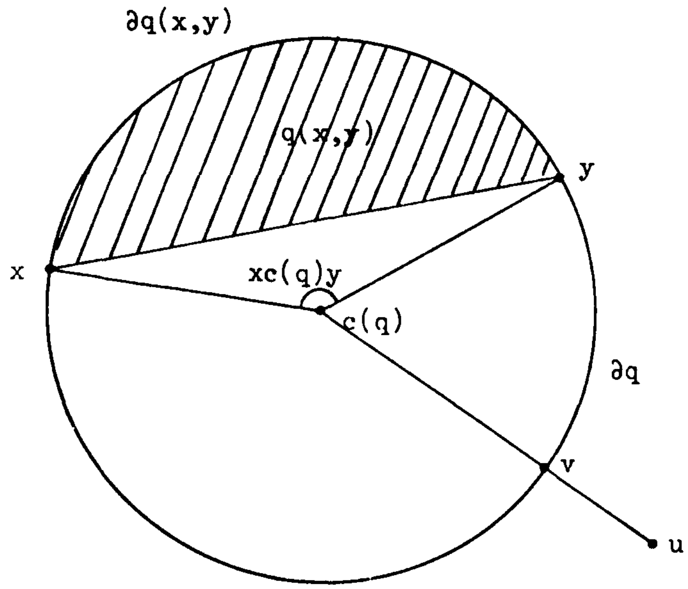

Figure 2.

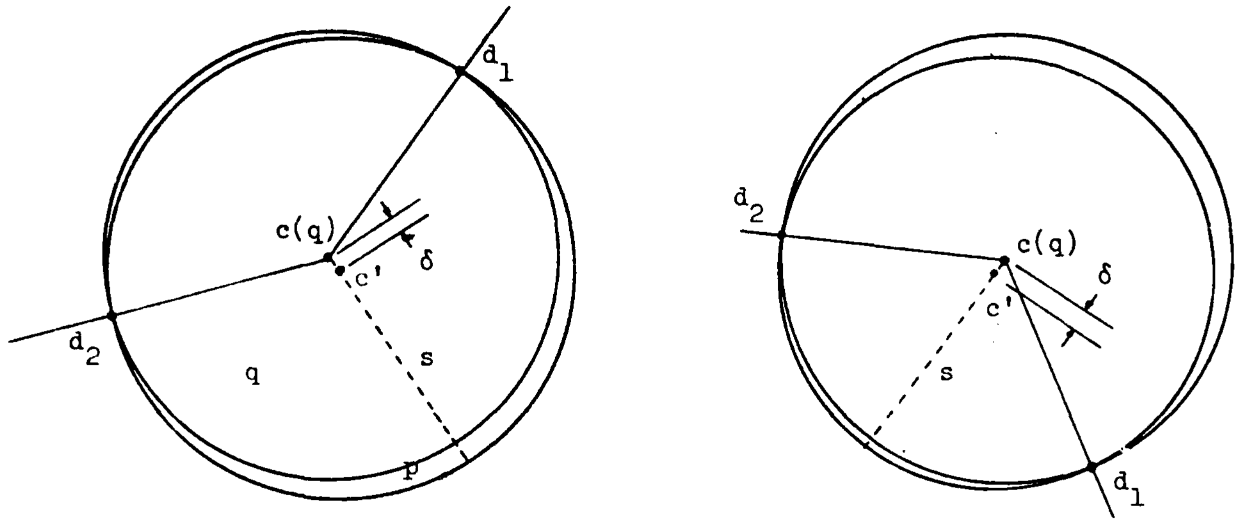

Figure 3.

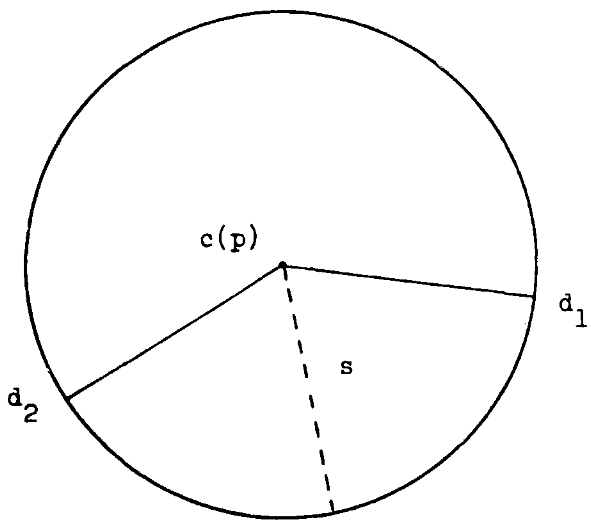

Figure 4.

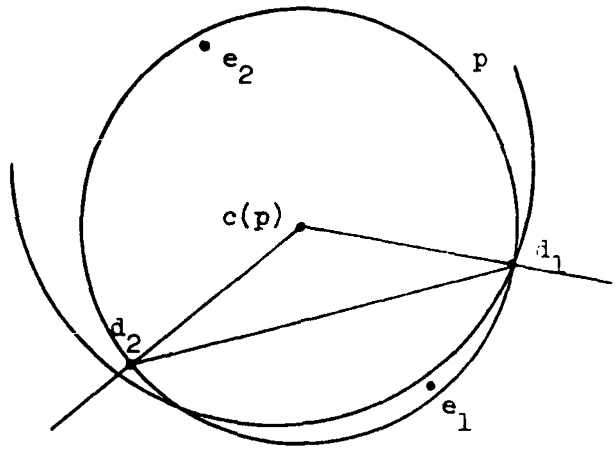

Figure 5.

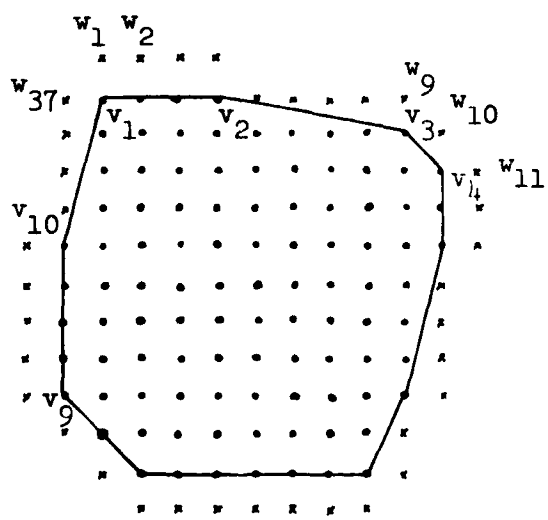

Figure 6. $P=\{v_1,v_2, \ldots ,v_{10}\}$ and $R= w_1,w_2, \ldots ,w_{37}$,
$P(v_1,v_3)=\{v_2\}$, $R(v_1,v_3)=\{w_1, w_2, \ldots ,w_{10}\}$,
$P(v_3,v_1)=\{v_4, \ldots ,v_{10}\}$ and $R(v_3,v_1)=\{w_{11}, \ldots ,w_{37}\}$.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Geometric properties of digital disks are discussed. An algorithm is presented that determines whether or not a given digital region is a digital disk.

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